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Classical Behavior After a Phase Transition: I. Classical Order Parameters

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We analyze the onset of classical field configurations after a phase transition. Firstly, we motivate the problem by means of a toy model in quantum mechanics. Subsequently, we consider a scalar field theory in which the system-field interacts with its environment, represented both by further scalar fields and by its own short-wavelength modes. We show that for very rapid quenches, the order parameter can be treated classically by the time taken for it to achieve its ground state values (spinodal time).

KEY WORDS: decoherence; phase transitions; topological defects.

1. INTRODUCTION

The emergence of classical behavior from a quantum system is a problem of interest in many branches of physics (Giulini *et al.*, 1996; Molmer, 1997a,b Paz and Zurek, 2000), from the foundations of quantum mechanics, condensed matter, and quantum optics, to quantum computing and quantum field theory. In the last few years, experimental evidence supports our theoretical understanding of the quantum to classical transition and also opens new avenues in the development of essential tools to understand the more hidden mysteries of the quantum world (Brune *et al.*, 1996; Friedman *et al.*, 2000; Monroe *et al.*, 1996; Myatt *et al.*, 2000; Rauschenbeutel *et al.*, 2000; Tesche, 2000; van del Wal *et al.*, 2000).

The quantum to classical transition, for a point particle, say, involves two different but very related conditions. The first one is that there should be *correlations*, i.e., the coordinates and momenta of a particle should be correlated in phase space, according to the classical equations of motion. For example, the Wigner function should have a peak at the classical trajectories. The second and

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equally important condition is the elimination of quantum interference between these classical trajectories, i.e., *decoherence*. Once decoherence eliminates interference terms (absent in the "classical world"), the Wigner function becomes a good candidate for a classical probability distribution. The generalization from particles to fields is straightforward, in principle, with the same attributes of correlations and decoherence.

The onset of classical behavior is a natural consequence of a quantum open system, triggered by the interaction between the system of interest and its environment. The coupling strength between system and bath sets the timescale after which we can consider our system as classical, according to both of the conditions we mentioned before. This temporal scale is usually called the *decoherence time* t_D. After this decoherence time we do not have macroscopic states in coherent superpositions anymore, and a probability distribution can be extracted that evolves by means of a generalized Fokker–Planck equation.

Our concern in this paper will be the quantum to classical transition of a single scalar order parameter field during continuous transitions, with the simplest double-well potential. This is an idealization of the phase transitions that are expected to occur at the GUT and EW scales in the standard big-bang cosmological model (Kolb and Turner, 1990).

An analysis of phase transitions in quantum field theory that takes the nonequilibrium nature of the dynamics into account from first principles has only recently begun to be addressed. In particular, the naive picture of a classical order parameter (inflaton or Higgs) field ϕ rolling down an adiabatic effective potential, that was once a mainstay of cosmological field theory modeling, has been shown to be suspect (Cormier and Holman, 2000). Alternatively, the suggestion by Kibble (1980) that while a nonadiabatic approach is essential, causality alone can set saturated bounds on time and distance scales during a transition, has been shown to be only partly true. The onset of classical behavior is absolutely crucial in the work of Kibble (1980) in that the experimental signal for the causal bound is the production of classical defects at a prescribed density. Topological defects are inevitable in most transitions, and they may have played a fundamental role in the formation of large scale structure (strings) (Zurek, 1985, 1996). Moreover, superabundance of some topological defects may contradict the observational evidence (magnetic monopoles).

In all the above-mentioned examples there is an order parameter that evolves from the false to the true vacuum of the theory: the Higgs fields in GUT and EW phase transitions, the inflaton field(s) in inflationary models, etc. Although these are quantum scalar fields with vanishing mean value (due to the symmetry of the initial quantum state), the order parameter is usually treated as a classical object. Our aim in this paper is to justify this assumption.

Our approach follows the analysis started by two of us in Lombardo and Mazzitelli (1996), where we studied the emergence of classical inhomogeneities from quantum fluctuations for a self-interacting quantum scalar field. We have

investigated there the decoherence induced on the long-wavelength field modes by coarse-graining the field modes with wavelength shorter than a critical value, in order to show how the system becomes classical because of the interaction with its environment (in that case composed of the short-wavelength field modes of the same field). For phase transitions the classicality of the order parameter can be analyzed along the same lines by extending the model to accommodate spontaneous symmetry breaking.

This is a difficult problem because, as has been pointed out in the literature, and as we will stress in what follows, nonperturbative and non-Gaussian effects are relevant in the analysis of phase transitions. As a trial run for this analysis we begin by presenting a toy model in which we will study the spread of a wave packet initially centered around the local maximum of a double-well potential, paying particular attention to the influence of the environment on the Wigner function and on the reduced density matrix. Only later we will extend our results to quantum field theory phase transitions.

The paper is organized as follows. In the next section we study the evolution of a wave packet initially centered on the top of a double-well potential. We describe the exact numerical evaluation of the evolution of the wave packet. We show that as the coupling between the system and the environment decreases, the decoherence time increases. Because of the nonlinearities of the potential, when the coupling vanishes there is no classical limit, not even classical correlations. In Section 3 we analyze the full problem in quantum field theory. Here, the situation is somewhat different. Although, again, the decoherence time increases as the coupling to the environment decreases, so does the spinodal time, the time for the order parameter to achieve its classical ground-state values. Thus, however weak the coupling, decoherence can still occur *before* the field has fallen to its classical level.

It is important to say that the issue of how the system evolves into the classical theory has been addressed in Habib *et al.* (1996) and Cooper *et al.* (1997). For some models, it has been shown that classicality emerges as a consequence of profuse particle creation. A nonperturbative large occupation number of long-wavelength modes produces, on average, a diagonal density matrix. This dephasing effect occurs at late times. Here we will consider a different model in which classicality is an early time event.

Finally, in Section 4, we present our final remarks and conclusions.

2. TOY MODEL: DOUBLE-WELL POTENTIAL AND ENVIRONMENT

The simplest field theory that permits a phase transition is that of a single real scalar field ϕ, with action

$$S[\phi] = \int d^4x \left\{ \frac{1}{2}\partial_\mu \phi \partial^\mu \phi + \frac{1}{2}\mu^2 \phi^2 - \frac{\lambda}{4!}\phi^4 \right\} \tag{1}$$

with ($\mu^2 > 0$) $\mathcal{Z}_2$ symmetry breaking. At high temperature the symmetry is restored. As we shall see later, a phase transition induced by a sudden temperature quench can be described by an effective field theory in which there is a change of sign in the mass term of the scalar field

$$S^{\rm eff}[\phi] = \int d^4x \left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2(t)\phi^2 - \frac{1}{4!}\lambda\phi^4\right], \tag{2}$$

where $m^2(t) = M^2 > 0$ for $t < 0$ and $m^2(t) = -\mu^2$ for t sufficiently positive. This change of sign in $m^2(t)$ breaks the global $\mathcal{Z}_2$ symmetry for positive t.

Understanding this transition, for even such a simple system, is difficult. As a preliminary exercise, we start by considering a toy model composed of a particle and a quantum anharmonic oscillator (the "system"), linearly coupled to an environment composed of an infinite set of harmonic oscillators. The total classical action is given by

$$\begin{aligned} S[x, q_n] &= S_{\rm syst}[x] + S_{\rm env}[q_n] + S_{\rm int}[x, q_n] \\ &= \int_0^t ds \left[\frac{1}{2}M\left(\dot{x}^2 - \Omega_0^2(t)x^2 - \frac{\lambda}{4}x^4\right)\right. \\ &\quad \left. + \sum_n \frac{1}{2}m_n\left(\dot{q}_n^2 - \omega_n^2 q_n^2\right)\right] - \sum_n C_n x q_n, \end{aligned} \tag{3}$$

where x and q_n are the coordinates of the particle and the oscillators, respectively. The quantum anharmonic oscillator is coupled linearly to each oscillator in the bath with strength C_n. In analogy with $m^2(t)$ of (1) we consider the simplest possible case of an *instantaneous* quench, in which $\Omega_0^2(t) = \Omega_0^2$ when $t < 0$ and $\Omega_0^2(t) = -\Omega_0^2$ when $t > 0$. The unstable particle, coordinate x, has an initial thermal distribution at temperature T, for which $\langle\hat{x}\rangle = 0$. For $t > 0$, it finds itself on the unstable hump of the potential and falls toward its stable minima at $\langle\hat{x}^2\rangle = 2\Omega_0^2/\lambda$.

2.1. The Environment

In the absence of an environment, and considering $\lambda = 0$, this situation has been discussed in detail by Guth and Pi (1991) and Kim and Lee (2000). In the presence of an environment the relevant objects for analyzing the quantum to classical transition in this model are the reduced density matrix and the associated Wigner function:

$$\rho_{\rm r}(x, x', t) = \int dq_n\ \rho(x, q_n, x', q_n, t), \tag{4}$$

and

$$W_{\rm r}(x,\,p,\,t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}dy\;e^{ipy}\;\rho_{\rm r}\left(x+\frac{y}{2},\,x-\frac{y}{2},\,t\right). \tag{5}$$

The reduced density matrix satisfies a closed master equation. This has been evaluated by Hu *et al.* (1993a,b) for the quantum Brownian motion problem with Ω_0^2 positive. Following the same procedure, we can write a corresponding master equation for the unstable particle (Zurek and Paz, 1994) by replacing Ω_0 by $i\Omega_0$ in the Hu–Paz–Zhang result

$$\begin{aligned} i\,\partial_t\rho_{\rm r}(x,\,x',\,t) &= \langle x|[H,\,\rho_{\rm r}]|x'\rangle \\ &\quad -i\,\gamma(t)(x-x')(\partial_x-\partial_{x'})\rho_{\rm r}(x,\,x',\,t) \\ &\quad +f(t)(x-x')(\partial_x+\partial_{x'})\rho_{\rm r}(x,\,x',\,t) \\ &\quad -i\,D(t)(x-x')^2\rho_{\rm r}(x,\,x',\,t). \end{aligned} \tag{6}$$

In (6), $H=H_{\rm syst}-\frac{1}{2}M\tilde{\Omega}^2(t)$, where $\tilde{\Omega}^2(t)$, renormalizes the natural frequency of the particle, $\gamma(t)$ is the dissipation coefficient, and $D(t)$ and $f(t)$ are the diffusion coefficients, which produce the decoherence effects. They depend on the properties of the environment as

$$\begin{aligned} \tilde{\Omega}^2(t) &= -\frac{2}{M}\int_0^t dt'\cosh(\Omega_0 t')\eta(t') \\ \gamma(t) &= -\frac{1}{2M\Omega_0}\int_0^t dt'\sinh(\Omega_0 t')\eta(t') \\ D(t) &= \int_0^t dt'\cosh(\Omega_0 t')\nu(t') \\ f(t) &= -\frac{1}{M\Omega_0}\int_0^t dt'\sinh(\Omega_0 t')\eta(t'), \end{aligned} \tag{7}$$

where $\eta(t)$ and $\nu(t)$ are the dissipation and noise kernels, respectively,

$$\begin{aligned} \eta(t) &= \int_0^\infty d\omega I(\omega)\sin\omega t \\ \nu(t) &= \int_0^\infty d\omega I(\omega)\coth\frac{\beta\omega}{2}\cos\omega t, \end{aligned}$$

and $I(\omega)=\mathcal{O}(C_n^2)$ is the spectral density of the environment.

The first term on the RHS of Eq. (6) gives the usual Liouville-like evolution; the term proportional to γ produces dissipation (γ is the relaxation coefficient). The term proportional to the diffusion coefficient $D(t)$, which is proportional to $(x-x')^2$ and positive definite, gives the main contribution to the decoherence

since it produces a diagonalization of the reduced density matrix. Let us write the reduced density matrix as

$$\rho_{\rm r}[x, x'; t] = G[x, x', t] \exp\left[-(x - x')^2 \int_0^t D(s)\, ds\right]. \tag{8}$$

Inserting this expression into the master equation it is easy to to see that the differential equation for $G[x, x', t]$ contains the usual Liouville-term plus additional contributions proportional to D, γ, and f. However, none of these additional terms is imaginary with the right sign for diffusion. An approximate solution of Eq. (6) is therefore (Zurek, 1994)

$$\rho_{\rm r}[x, x'; t] \approx \rho_{\rm r}^{\rm u}[x, x', t] \exp\left[-(x - x')^2 \int_0^t D(s)\, ds\right], \tag{9}$$

where $\rho_{\rm r}^{\rm u}$ takes into account the unitary evolution.

Alternatively, one can derive the following evolution equation for the reduced Wigner function of the system (Paz *et al.*, 1993):

$$\begin{aligned}\dot{W}_{\rm r}(x, p, t) = \{H_{\rm syst}, W_{\rm r}\}_{\rm PB} - \frac{\lambda}{4} x \partial^3_{ppp} W_{\rm r} + 2\gamma(t)\partial_p(p W_{\rm r}) \\ + D(t)\partial^2_{pp} W_{\rm r} - f(t)\partial^2_{px} W_{\rm r}.\end{aligned} \tag{10}$$

Let us concentrate on the evolution equation (10). The first term on the right-hand side of Eq.(10) is the Poisson bracket, corresponding to the usual classical evolution. The second term includes the quantum correction (we have set $\hbar = 1$). The last three terms describe dissipation and diffusion effects due to coupling to the environment. In order to simplify the problem, we consider a high-temperature ohmic ($I(\omega) \sim \omega$) environment. In this approximation the coefficients in Eq. (10) become constants: $\gamma(t) = \gamma_0$, $f \sim 1/T$, and $D = 2\gamma_0 k_{\rm B} T$. The normal diffusion coefficient D is the term responsible for decoherence effects and at high temperatures is much larger than γ_0 and f. Therefore, in Eq. (10), we may neglect the dissipation and the anomalous diffusion terms in comparison to the normal diffusion. As we saw in (9), it is the diffusion exponential in this equation that enforces the approximate diagonalization of $\hat{\rho}$ in this coordinate basis.

It is important to note that this high-temperature approximation is well defined after a timescale of the order of $1/(k_{\rm B}T) \sim \gamma_0/D$ (with $\hbar = 1$). The relevant regime of evolution for our systems takes place at times comfortably larger than this timescale.

2.2. Numerical Analysis

We have solved Eq. (10) numerically, in the high-temperature limit, for different values of the diffusion coefficient D, in order to illustrate its relevance in

the quantum to classical transition. Details are given elsewhere (Lombardo *et al.*, 2000). We have chosen as initial condition a Gaussian state centered at $x_0 = p_0 = 0$ with minimal uncertainty ($\sigma_x^2 = 0.5$ and $\sigma_p^2 = 0.5$). The Wigner function is initially positive definite, and different from zero only near the top of the potential. We have set the coupling constant $\lambda = 0.01$, the renormalized frequencies $\tilde{\Omega} = \omega_n = 1$ (we are measuring time in units of $\tilde{\Omega}$) and the bare masses also equal to one.

It is illustrative to examine first the exact result when the environment is absent (for this case we have solved numerically the Schrödinger equation). Initially the Gaussian Wigner function begins to squeeze in the $x = p$ direction, and before the spinodal time ($t_{\rm sp} \sim 2.3$) it becomes a nonpositive function (Fig. 1). Our definition of the spinodal time is that time at which $\langle \hat{x}^2 \rangle_t = 2\Omega_0^2/\lambda$, its symmetry-breaking value. During the evolution, the Wigner function covers all the phase space (Fig. 2) and it is clear that it is not possible to consider it as a classical probability distribution. Although we started with a special initial state (Gaussian with minimum uncertainty), the nonlinearities of the potential make the Wigner function a nonpositive distribution.

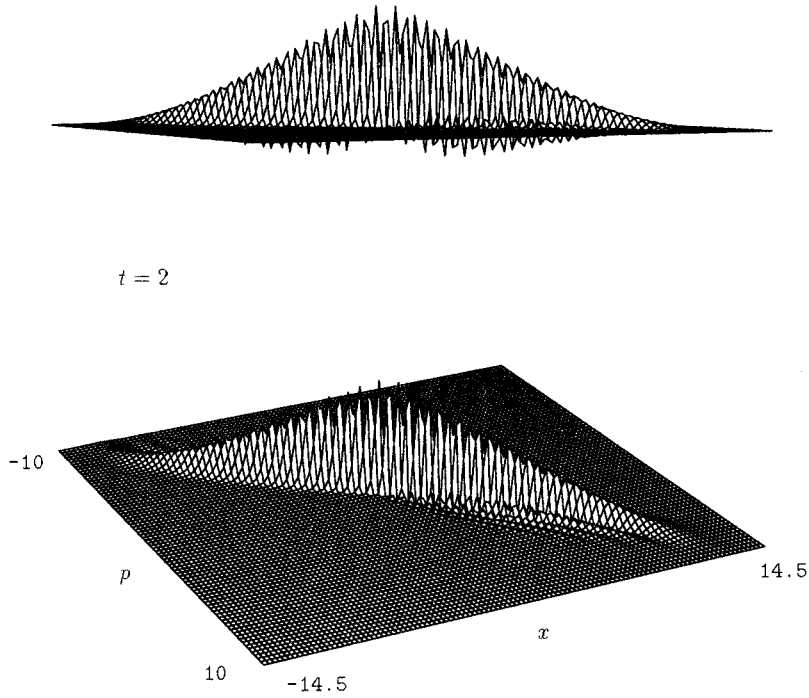

Fig. 1. The Wigner function for $t = 2 < t_{\rm sp} = 2.3$, when no environment is considered.

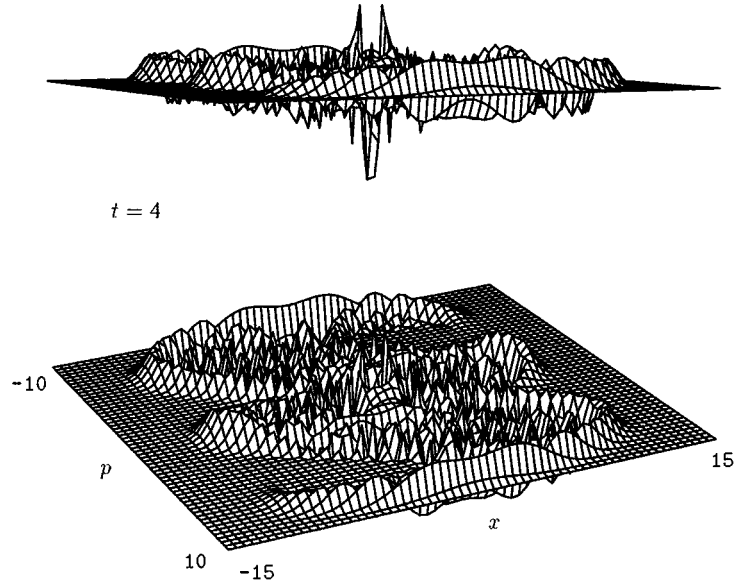

Fig. 2. The Wigner function for $t = 4$. As the function is not positive definite, there is no classical correlation.

Let us now consider a coupling with an environment such that the normal diffusion coefficient is $D = 0.01$. As expected, the evolution of the Wigner function is similar to the previous one at early times (Figs. 3 and 4). However, as can be seen from Figs. 5 and 6, at long times it becomes positive definite and peaked around the classical phase space.

The effect of the environment is more dramatic for larger values of the diffusion coefficient. In our last example, $D = 1$, the quantum to classical transition takes place almost instantaneously, even before the quantum particle pass through the spinodal point (Figs. 7 and 8).

It is interesting to note that as the diffusion coefficient grows, the amplitude of the Wigner function decreases. This is due to the fact that the decoherence increases with D. The reduced density matrix diagonalizes. As a consequence, its 'Fourier transform,' the reduced Wigner function, spreads out.

Our numerical results show explicitly that the existence of the environment is crucial in the quantum to classical transition. The decoherence time depends on the temperature and the coupling between system and environment through the diffusion coefficient D (Zurek, 1986).

3. PHASE TRANSITIONS IN FIELD THEORY

We can now tackle the full quantum ϕ field theory of (1). A simplified analysis of the model has been given elsewhere (Lombardo *et al.*, 2001). As before, the

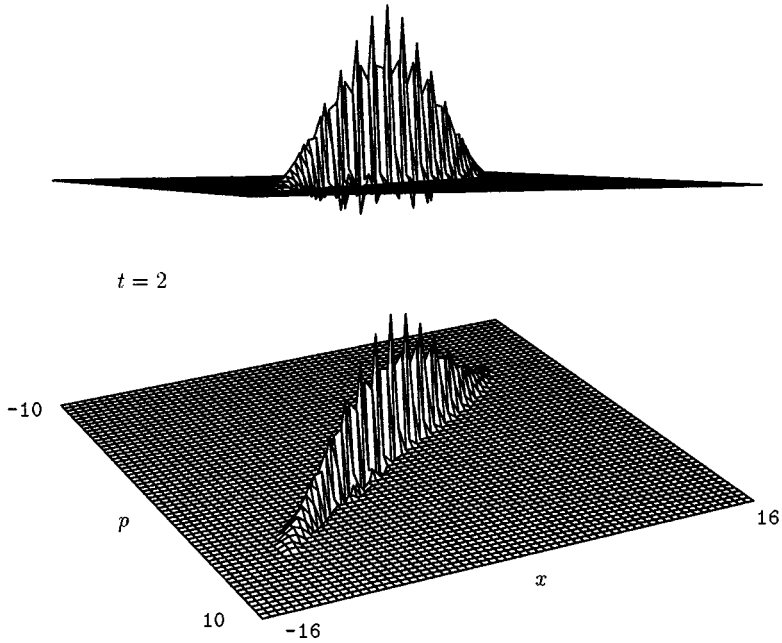

Fig. 3. The Wigner function for $t = 2$, including an environment with diffusion coefficient $D = 0.01$.

onset of classical behavior is due to the environment. For the infinite degree of freedom quantum field, the field ordering after the transition begins is due to the growth in amplitude of its unstable long-wavelength modes. For these modes the environment consists of the short-wavelength modes of the field, together with all the other fields χ_a. with which the ϕ inescapably interacts (Lombardo and Mazzitelli, 1996). The inclusion of explicit environment fields χ_a both reflects on the fact that a scalar field in isolation is physically unrealistic, as well as provides us with a systematic approximation scheme.

Although the system field can never avoid the decohering environment of its short-wavelength modes, to demonstrate the effect of an environment we first consider the case in which it is taken to be composed only of the fields χ_a. The short-wavelength modes of the ϕ field will be considered afterwards. Specifically, we take the simplest scalar classical action

$$S[\phi, \chi] = S_{\text{syst}}[\phi] + S_{\text{env}}[\chi] + S_{\text{int}}[\phi, \chi], \tag{11}$$

$$S_{\text{syst}}[\phi] = \int d^4x \left\{ \frac{1}{2}\partial_\mu \phi \partial^\mu \phi + \frac{1}{2}\mu^2\phi^2 - \frac{\lambda}{4!}\phi^4 \right\},$$

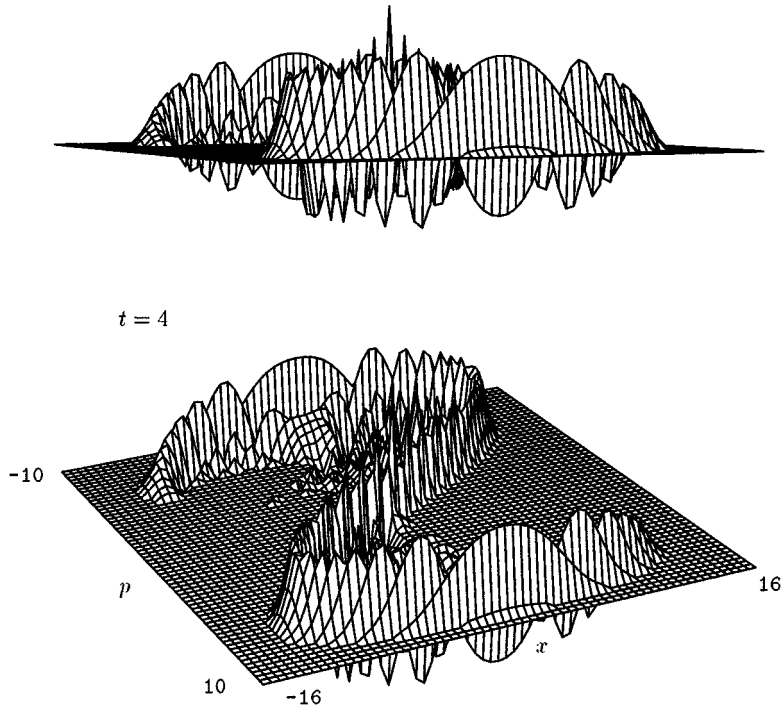

Fig. 4. The same Wigner function (as in Fig. 3) for time $t = 4$. It is not positive definite.

$$S_{\rm env}[\chi_a] = \sum_{a=1}^{N} \int d^4x \left\{ \frac{1}{2}\partial_\mu \chi_a \partial^\mu \chi_a - \frac{1}{2} m_a^2 \chi_a^2 \right\},$$

$$S_{\rm int}[\phi, \chi] = -\sum_{a=1}^{N} \frac{g_a}{8} \int d^4x \phi^2(x) \chi_a^2(x), \tag{12}$$

where $\mu^2, m^2 > 0$.

We see that the quantum mechanics action of (3) is, in large part, a simplified version of (11). To make any progress it is important that the environment be as simple as possible. To that end we have included no explicit self-interaction of the χ_a fields, which interact through the ϕ field as intermediary. However, there is one significant difference in that the interactions of the ϕ field with the environment in (12) are *quadratic*, and not linear as in (3). The more conventional choice of a linear coupling to the environment was made in the previous section (and Lombardo *et al.*, 2000) to give a model directly comparable to similar particle models with no symmetry breaking, for which much work has been done. Although it has been adopted for quantum field theories (Kim and Lee, 2000) a linear–linear coupling

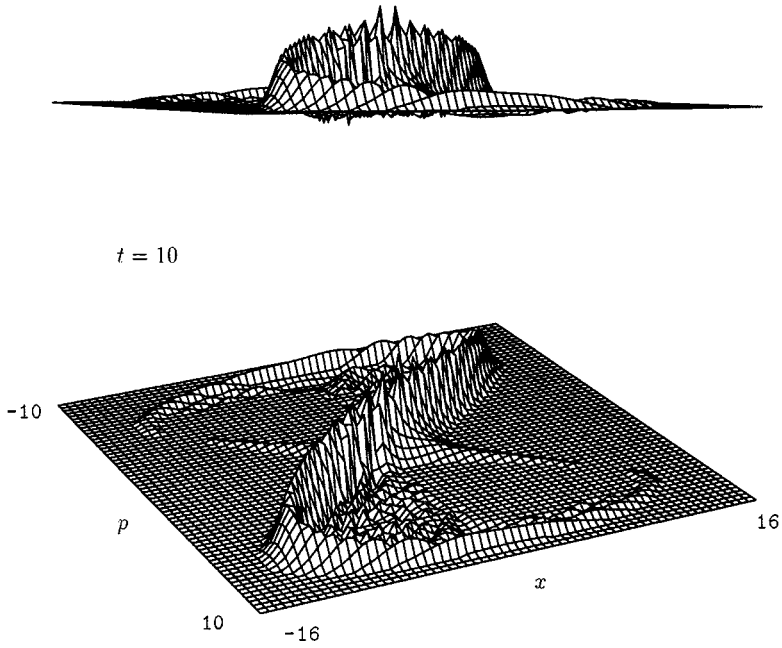

Fig. 5. The same Wigner function (as in Fig. 3) for $t = 10$.

is inappropriate and we have taken the simplest quadratic–quadratic couplings in S_{int} for convenience. The extension to Yukawa couplings is straightforward, and will be given elsewhere. We are reminded that for nonlinear couplings like $x^n q_{\text{i}}^m$ in the quantum mechanical problem, one expects the master equation to contain terms of the form $i D^{(n,m)}(t)(x^n - x'^n)^2 \rho_{\text{r}}$. We shall find a similar effect here. The nonlinear coupling to the environment is crucial to our conclusions.

Further, in our present model, the environment has a strong impact upon the system-field, but not vice versa, whenever possible. The simplest way to implement this is to take a large number $N \gg 1$ of χ_a fields with comparable masses $m_a \simeq \mu$ weakly coupled to the ϕ, with λ; $g_a \ll 1$. Thus, at any step, there are N weakly coupled environmental fields influencing the system field, but only one system field to back-react upon the explicit environment.

3.1. Initial Conditions

Before we examine the model in detail, there are some general observations to be made about initial conditions and the way in which the transition is implemented. Like any simple scalar theory the model displays a continuous transition at a

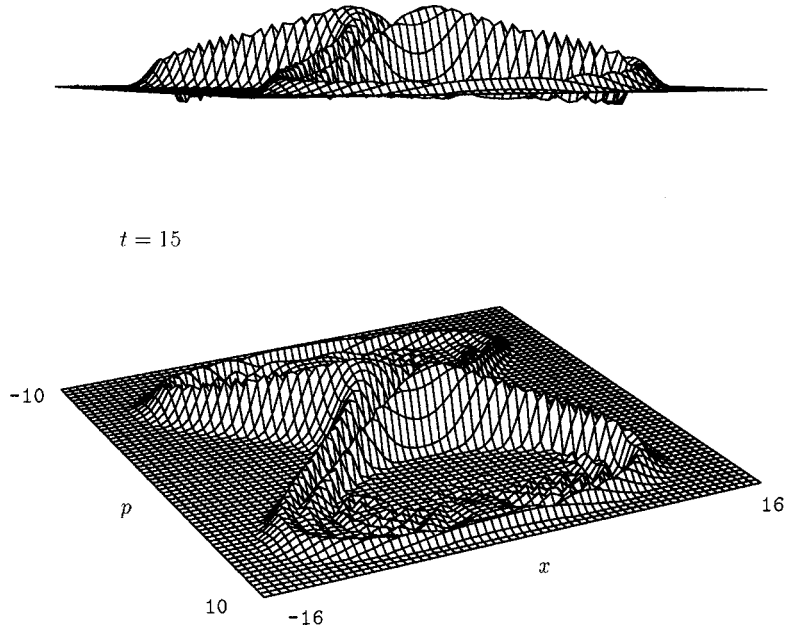

Fig. 6. The same Wigner function, (as in Fig. 3) for $t = 15$. Only for $t \gg t_{sp}$ do we have classical correlations and a positive Wigner function.

temperature T_c,

$$T_c^2 = \frac{\mu^2}{\lambda + \sum g_a} \gg \mu^2.$$

The environmental fields χ_a reduce T_c, and in order that $T_c^2 \gg \mu^2$, we must take

$$\lambda + \sum g_a \ll 1.$$

For order of magnitude estimations it is sufficient to take identical $g_a = \bar{g}/\sqrt{N}$, whereby

$$1 \gg 1/\sqrt{N} \gg \bar{g} \simeq \lambda,$$

i.e. the χ_a "tadpole" diagrams completely overwhelm the ϕ self-interaction tadpole diagram in generating the ϕ thermal mass.

With $\eta = \sqrt{6\mu^2/\lambda}$ determining the position of the minima of the potential and the final value of the order parameter, this choice of coupling and environments

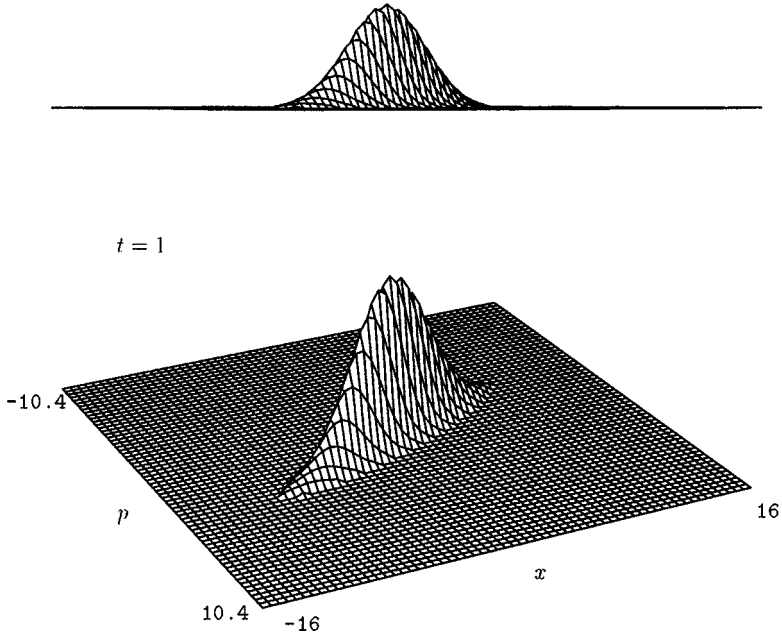

Fig. 7. The early time ($t = 1$) Wigner function for a diffusion coefficient $D = 1$.

gives the hierarchy of scales

$$\mu^2 \ll T_c^2 = O\left(\frac{\eta^2}{\sqrt{N}}\right) \ll \eta^2,$$

important in establishing a reliable approximation scheme. Further, with this choice the dominant hard loop contribution of the ϕ field to the χ_a thermal masses is

$$\delta m_T^2 = O(\bar{g} T_c^2/\sqrt{N}) = O(\mu^2/N) \ll \mu^2.$$

Similarly, the two-loop (setting sun) diagram, which is the first to contribute to the discontinuity of the χ-field propagator, is of magnitude

$$\bar{g}^2 T_c^2/N = O(g\mu^2/N^{3/2}) \ll \delta m_T^2,$$

in turn. That is, the effect of the thermal bath on the propagation of the environmental χ fields is ignorable.

This was our intention in model-making: to construct an environment that reacted on the system field, but was not reacted upon by it to any significant extent. We stress that this is not a Hartree or large-N approximation of the type that, to date, has been the standard way to proceed (Boyanovsky *et al.*, 1994, 1995; Cooper *et al.*, 1997; Ramsey and Hu, 1997) for a *closed* system.

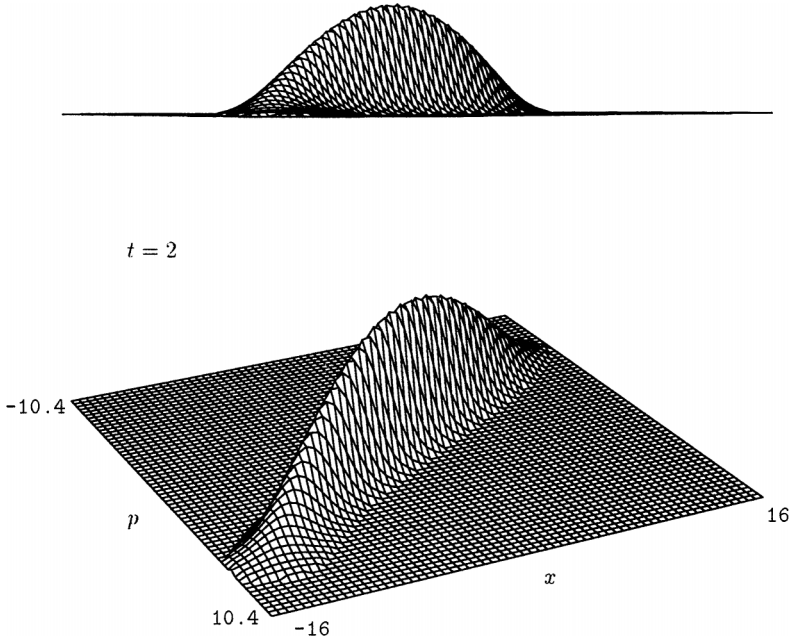

Fig. 8. The same as Fig. 7 for $t = 2$. Classicality emerges before the spinodal time.

We shall assume that the initial states of the system and environment are both thermal, at a temperature $T_0 > T_c$. We then imagine a change in the global environment (e.g. expansion in the early universe) that can be characterized by a change in temperature from T_0 to $T_f < T_c$. That is, we do not attribute the transition to the effects of the environment-fields. On incorporating the hard thermal loop tadpole diagrams of the χ (and ϕ) fields in the ϕ mass term leads to the effective action for ϕ quasiparticles,

$$S^{\rm eff}_{\rm syst}[\phi] = \int d^4x \left\{ \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2_\phi(T_0)\phi^2 - \frac{\lambda}{4!}\phi^4 \right\},$$

where $m^2_\phi(T_0) = -\mu^2(1 - T_0^2/T_c^2) = M^2 > 0$. As a result, we can take an initial factorized density matrix at temperature T_0 of the form $\hat{\rho}[T_0] = \hat{\rho}_\phi[T_0]\hat{\rho}_\chi[T_0]$, where $\hat{\rho}_\phi[T_0]$ is determined by the quadratic part of $S^{\rm eff}_{\rm syst}[\phi]$ and $\hat{\rho}_\chi[T_0]$ by $S_{\rm env}[\chi_a]$. Yet again, the many χ_a fields have a large effect on ϕ, but the ϕ field has negligible effect on the χ_a. Provided the change in temperature is not too slow, the exponential instabilities of the ϕ field grow so fast that the field has populated the degenerate vacua well before the temperature has dropped to zero. Since the temperature T_c has no particular significance for the environment field, for these early times we can keep the temperature of the environment

fixed at $T_\chi = T_0 = \mathcal{O}(T_c)$ (our calculations are only at the level of orders of magnitude).

Since it is the system-field ϕ whose behavior changes dramatically on taking T_ϕ through T_c, in this paper we adopt an *instantaneous* quench for T_ϕ from T_0 to $T_f = 0$ at time $t = 0$, in which $m_\phi^2(T)$ changes sign and magnitude instantly, concluding with the value $m_\phi^2(t) = -\mu^2$, $t > 0$ that we cited in (1). An *instantaneous* quench is sufficient to demonstrate the rapidity with which the environment forces the system field to become classical. Meanwhile, for simplicity the χ_a masses are fixed at the common value $m \simeq \mu$.

3.2. Tracing out the χ Fields

At time $t > 0$, the reduced density matrix $\rho_r[\phi^+, \phi^-, t] = \langle \phi^+ | \hat{\rho}_r(t) | \phi^- \rangle$ is now

$$\rho_r[\phi^+, \phi^-, t] = \int \mathcal{D}\chi_a \, \rho[\phi^+, \chi_a, \phi^-, \chi_a, t],$$

where $\rho[\phi^+, \chi_a^+, \phi^-, \chi_a^-, t] = \langle \phi^+ \chi_a^+ | \hat{\rho}(t) | \phi^- \chi_a^- \rangle$ is the full density matrix. Since we would like to be able to distinguish between different classical system-field configurations evolving after the transition, we will only be interested in the field-configuration basis for this reduced density matrix (in analogy with the quantum Brownian motion model of the previous section). The environment will have had the effect of making the system essentially classical once $\rho_r(t)$ is, effectively, diagonal. [Our earlier comments on dephasing remain valid, and we stress again that our understanding of what constitutes classical behavior is essentially different from the dephasing effects found in Habib *et al.* (1996) and Cooper *et al.* (1997).] Quantum interference can then be ignored and the system is said to have decohered. At the same time, we obtain a classical probability distribution from the diagonal part of $\rho_r(t)$, or equivalently, by means of the reduced Wigner function. For weak coupling there will be no 'recoherence' at later times in which the sense of classical probability will be lost.

Its temporal evolution is given by

$$\rho_r[\phi_f^+, \phi_f^-, t] = \int d\phi_i^+ \int d\phi_i^- \, J_r[\phi_f^+, \phi_f^-, t \mid \phi_i^+, \phi_i^-, t_0] \;\; \rho_r[\phi_i^+ \phi_i^-, t_0],$$

where J_r is the reduced evolution operator

$$J_r[\phi_f^+, \phi_f^-, t \mid \phi_i^+, \phi_i^-, t_0] = \int_{\phi_i^+}^{\phi_f^+} \mathcal{D}\phi^+ \int_{\phi_i^-}^{\phi_f^-} \mathcal{D}\phi^- \; e^{i\{S[\phi^+] - S[\phi^-]\}} F[\phi^+, \phi^-]. \tag{13}$$

The Feynman–Vernon influence functional $F[\phi^+, \phi^-]$ (Feynman and Vernon, 1963) is defined as

$$F[\phi^+,\phi^-] = \int d\chi_{\rm ai}^+ \int d\chi_{\rm ai}^-\ \rho_\chi[\chi_{\rm ai}^+,\chi_{\rm ai}^-,t_0]\int d\chi_{\rm af}$$
$$\times \int_{\chi_{\rm ai}^+}^{\chi_{\rm af}} \mathcal{D}\chi_a^+ \int_{\chi_{\rm ai}^-}^{\chi_{\rm af}} \mathcal{D}\chi_a^- \exp\left(i\{S_{\rm env}[\chi_a^+]+S_{\rm int}[\phi^+,\chi_a^+]\right)$$
$$\times \exp\left(-i\{S_{\rm env}[\chi_a^-]+S_{\rm int}[\phi^-,\chi_a^-]\}\right).$$

Beginning from this initial distribution, peaked around $\phi = 0$, we follow the evolution of the system under the influence of the environment fields, with Hamiltonian determined from (11). From the influence functional we define the influence action $\delta A[\phi^+,\phi^-]$ as

$$F[\phi^+,\phi^-] = \exp i\,\delta A[\phi^+,\phi^-]. \tag{14}$$

The total coarse-grained effective action is

$$A[\phi^+,\phi^-] = S[\phi^+] - S[\phi^-] + \delta A[\phi^+,\phi^-].$$

We will calculate the influence action to one loop (two vertices) for large N using closed-time path correlators. It is the imaginary part that contains the information about the onset of classical behavior. If $\Delta = \frac{1}{2}(\phi^{+2} - \phi^{-2})$, we find

$$\mathrm{Im}\,\delta A = -\frac{\bar{g}^2}{16}\int d^4x \int d^4y\ \Delta(x) N_{\rm q}(x,y)\Delta(y). \tag{15}$$

In (15), $N_{\rm q}(x-y) = \mathrm{Re}\,G_{++}^2(x,y)$ is the noise (diffusion) kernel, where G_{++} is the relevant closed-timepath correlator of the χ field at temperature T_0. Nonleading one-loop terms are smaller by a factor of $\mathcal{O}(N^{-1/2})$.

The first step in the evaluation of the master equation is the calculation of the density matrix propagator $J_{\rm r}$ from Eq. (13). In order to estimate the functional integration that defines the reduced propagator, we perform a saddle point approximation

$$J_{\rm r}[\phi_{\rm f}^+,\phi_{\rm f}^-,t \mid \phi_{\rm i}^+,\phi_{\rm i}^-,t_0] \approx \exp i\,A[\phi_{\rm cl}^+,\phi_{\rm cl}^-],$$

where $\phi_{\rm cl}^\pm$ is the solution of the equation of motion $\frac{\delta \mathrm{Re}A}{\delta\phi^+}\mid_{\phi^+=\phi^-}=0$ with boundary conditions $\phi_{\rm cl}^\pm(t_0)=\phi_{\rm i}^\pm$ and $\phi_{\rm cl}^\pm(t)=\phi_{\rm f}^\pm$. It is very difficult to solve this equation analytically. For simplicity, we assume that the system-field contains only one Fourier mode with $\vec{k}=\vec{k}_0$. We are motivated in this by the observation that the long-wavelength modes start growing exponentially as soon as the quench is performed and rapidly dominate the fluctuation power spectrum (Boyanovsky *et al.*, 1994, 1995; Ramsey and Hu, 1997). Modes with $|k_0|^2 > \mu^2$ will oscillate.

Further, we are interested primarily in the order parameter

$$\phi(t) = \phi(k_0=0,t) = \lim_{V\to\infty}\frac{1}{V}\int_{\mathbf{x}\in V} d^3x\,\phi(\mathrm{x},t),$$

and all our subsequent calculations will be for theis zero-frequency mode. We write the spatially constant classical solution as $\phi(s) = f(s, t)$, where $f(s, t)$ satisfies the boundary conditions $f(0, t) = \phi_i$ and $f(t, t) = \phi_f$. Qualitatively, $f(s, t)$ grows exponentially with s for $t \leq t_{sp}$, and oscillates for $t_{sp} < s < t$ when $t > t_{sp}$. We shall therefore approximate its time dependence for $t \leq t_{sp}$ as

$$f(s, t) = \phi_i u_1(s, t) + \phi_f u_2(s, t), \tag{16}$$

where $u_1(0, t) = 1$, $u_1(t, t) = 0$ and $u_2(0, t) = 0$, $u_2(t, t) = 1$, with solution

$$u_1(s, t) = \frac{\sinh[\mu(t - s)]}{\sinh(\mu t)}, \qquad u_2(s, t) = \frac{\sinh(\mu s)}{\sinh(\mu t)}.$$

We shall justify the use of the linear equation later. In fact, we shall see then that it is not an unreasonable approximation until almost t_{sp}.

In order to solve the master equation we must compute the final time derivative of the propagator J_r, and after that eliminate the dependence on the initial field configurations $\phi_i^{\pm}$ coming from the classical solutions $\phi_{cl}^{\pm}$. This is the usual procedure; see Lombardo and Mazzitelli (1996).

As we are solely interested in the onset of classical behavior, it is sufficient to calculate the correction to the usual unitary evolution coming from the noise kernel. For clarity we drop the suffix f on the final state fields. If $\Delta = (\phi^{+2} - \phi^{-2})/2$ for the *final* field configurations, then the master equation for $\rho_r(\phi^+, \phi^-, t)$ is

$$i\dot{\rho}_r = \langle \phi^+ | [H, \hat{\rho}_r] | \phi^- \rangle - i\frac{\overline{g}^2}{16} V \Delta^2 D(t) \rho_r + \cdots \tag{17}$$

The presence of Δ in (17), rather than $\phi_+ - \phi_-$, is a consequence of the quadratic coupling to the environment in S_{int}. Since our main concern is with the diagonalization of ρ_r, it is not necessary to consider the evolution equation for the Wigner function, as has been shown in the literature of quantum Brownian motion (Lombardo *et al.*, 2000; Paz *et al.*, 1993). The volume factor V that appears in the master equation is due to the fact that we are considering a density matrix which is a functional of two different field configurations, $\phi^{\pm}(\vec{x}) = \phi^{\pm}$, which are spread over all space. The-time dependent diffusion coefficient $D_\chi(t)$ due to each of the many external environmental χ fields is given by

$$D_\chi(t) = 3 \int_0^t ds\, u(s, t)\, \mathrm{Re}\, G_{++}^2(0; t - s), \tag{18}$$

where

$$u(s, t) = \left[u_2(s, t) - \frac{\dot{u}_2(t, t)}{\dot{u}_1(t, t)} u_1(s, t) \right]^2. \tag{19}$$

For the case of an instantaneous quench, $u(s, t) = \cosh^2 \mu(t - s)$ when $t \leq t_{sp}$, and is an oscillatory function of time when $t > t_{sp}$.

Although G_{++} is oscillatory at all times, the exponential growth of $u(t)$ enforces a similar behavior on $D_\chi(t)$. In the high-temperature limit ($k_B T \gg \mu$), the explicit expression for that contribution to the diffusion coefficient due to the χ_a fields alone is

$$D_\chi(t) = \frac{5(k_B T)^2}{64\pi^2} \int_0^t ds\, u(s,t) \int_0^\infty dp\, \frac{p^2}{\omega^4} \cos[2\omega s], \tag{20}$$

where $\omega^2 = p^2 + m^2$ and we have set $m^2 = \mu^2$.

For times $\mu t \gg 1$, the integration in (20) is dominated by the behavior at $s = 0$:

$$D_\chi(t) \sim (k_B T_0/\mu)^2\, u(0,t) \sim (k_B T_0/\mu)^2\, \exp[2\mu t]. \tag{21}$$

The spinodal time t_{sp}, is again defined as the time for which $\langle \phi^2 \rangle_t \sim \eta^2 = 6\mu^2/\lambda$. For $t > t_{sp}$, the diffusion coefficient stops growing and oscillates around $D(t = t_{sp})$.

3.3. Short-Wavelength Modes

In our present model the environment fields χ_a are not the only decohering agents. The environment is also constituted by the short-wavelength modes of the self-interacting field ϕ. Therefore, we split the field as $\phi = \phi_< + \phi_>$, where the system-field $\phi_<$ contains the modes with wavelengths longer than the critical value μ^{-1}, while the bath or environment-field contains wavelengths shorter than μ^{-1}. This gives an additional one-loop contribution $D_\phi(t)$ to the diffusion function with the same $u(s,t)$ but a G_{++} constructed form the short-wavelength modes of the ϕ field as it evolves from the top of the potential hill. Without the additional powers of N^{-1} to order contributions, summation of loop diagrams is essential to get a reliable G_{++}. However, it is not necessary to calculate $D_\phi(t)$ in order to get a good estimate of t_D. Since the contribution of $D_\phi(t)$ to the overall diffusion function is positive we can derive an *upper* bound on the decoherence time t_D from the reliable diffusion functions D_χ alone.

However, we would not expect the inclusion of the ϕ field to give a qualitative change. Specifically, we note that, if we take ϕ field propagators dressed by only the simplest tadpole diagrams and ignore two-loop self-interaction diagrams, then the diffusion correction due to the ϕ loop is now similar to that of the χ loops. The effect is that the short-wavelength modes in the one-loop diagrams from which they are calculated have been kept at the initial temperature T_0, on the grounds that passing through the transition quickly has no effect on them. The quench mimics a slower evolution of temperature in which only the long-wavelength modes show instabilities that the transition induces. That is, with $\bar{g} \simeq \lambda$ and no $1/N$ factor, the short-wavelength ϕ modes would have the same effect on the dissipation, qualitatively, as *all* the explicit environmental fields put together. However, at an

order of magnitude level there is no change, since the effect is to replace $\bar{g}^2$ by $\bar{g}^2 + O(\lambda^2) = O(\bar{g}^2)$.

3.4. The Decoherence Time

Using the positivity of D_ϕ we estimate the decoherence time t_D for the model by considering the approximate solution to the master equation (17),

$$\rho_r[\phi_<^+, \phi_<^-; t] \lesssim \rho_r^u[\phi_<^+, \phi_<^-; t] \exp\left[- V\Gamma \int_0^t ds\, D_\chi(s)\right],$$

where ρ_r^u is the solution of the unitary part of the master equation (i.e. without environment). It is obvious from this (and also from (17)) that the diagonal density matrix just evolves like the unitary matrix (the environment has almost no effect on the diagonal part of ρ_r). In terms of the dimensionless fields $\bar{\phi} = (\phi_<^+ + \phi_<^-)/2\mu$, and $\delta = (\phi_<^+ - \phi_<^-)/2\mu$, we have $\Gamma = (1/16)\bar{g}^2\mu^4\bar{\phi}^2\delta^2$.

The system behaves classically when ρ_r is appropriately diagonal. We therefore look at the ratio

$$\left|\frac{\rho_r[\bar{\phi}+\delta, \bar{\phi}-\delta; t]}{\rho_r[\phi, \phi; t]}\right| \lesssim \left|\frac{\rho_r^u[\bar{\phi}+\delta, \bar{\phi}-\delta; t]}{\rho_r^u[\bar{\phi}, \bar{\phi}; t]}\right| \exp\left[- V\Gamma \int_0^t ds\, D_\chi(s)\right]. \quad (22)$$

It is not possible to obtain an analytic expression for the ratio of unitary density matrices that appears in Eq. (22). The simplest approximation is to neglect the self-coupling of the system field (Guth and Pi, 1991). In this case the unitary density matrix remains Gaussian at all times as

$$\left|\frac{\rho_r^u[\bar{\phi}+\delta, \bar{\phi}-\delta; t]}{\rho_r^u[\phi, \phi; t]}\right| = \exp[-\frac{T_c}{\mu}\delta^2 p^{-1}(t)], \quad (23)$$

where $p^{-1}(t)$, essentially $\mu^2\langle\phi^2\rangle_t^{-1}$, decreases exponentially with time to a value $\mathcal{O}(\lambda)$. This approximation can be improved by means of a Hartree-like approximation (Boyanovsky *et al.*, 1994, 1995; Ramsey and Hu, 1997). In this case the ratio is still given by Eq. (23), but now $p^{-1}(t)$ decreases more slowly as t approaches t_{sp}. In any case, in the unitary part of the reduced density matrix the nondiagonal terms are not suppressed. Therefore, in order to obtain classical behavior, the relevant part of the reduced density matrix is the term proportional to the diffusion coefficient in Eq. (22), since it is this that enforces its diagonalization.

The decoherence time t_D sets the scale after which we have a classical system-field configuration. According to our previous discussion, it can be defined as the solution to

$$1 \gtrsim V\Gamma \int_0^{t_D} ds\, D(s). \quad (24)$$

It corresponds to the time after which we are able to distinguish between two different field amplitudes (inside a given volume V).

Suppose we reduce the couplings $\bar{g}$, λ of the system ϕ field to its environment. Since, as a one-loop construct, $\Gamma \propto \bar{g}^2$, λ^2 our first guess would be that as $\bar{g}$, λ decrease, t_D increases and the system takes longer to become classical. This is not really the case. The reason is twofold. First, there is the effect that $\Gamma \propto T_0^2$, and $T_0^2 \propto \lambda^{-1}$ is nonperturbatively large for a phase transition. Second, because of the nonlinear coupling to the environment, obligatory for quantum field theory, $\Gamma \propto \bar{\phi}^2$. The completion of the transition finds $\bar{\phi}^2 \simeq \eta^2 \propto \lambda^{-1}$ also nonperturbatively large. This suggests that Γ, and hence t_D, can be independent of λ. The situation would be different for a linear coupling to the environment, for which $\bar{\phi}^2$ would not be present, or a cold initial state in which ϕ is peaked about $\phi = 0$. In fact, the situation is a little more complicated, but the corollary that t_D does not exceed $t_{\rm sp}$ as the couplings become weaker remains true.

In order to quantify the decoherence time we have to fix the values of V, δ, and $\bar{\phi}$. V is understood as the minimal volume inside which we do not accept coherent superpositions of macroscopically distinguishable states for the field. Thus, our choice is that this volume factor is $\mathcal{O}(\mu^{-3})$ since μ^{-1} (the Compton wavelength) is the smallest scale at which we need to look. Inside this volume, we do not discriminate between field amplitudes which differ by $\mathcal{O}(\mu)$, and therefore we take $\delta \sim \mathcal{O}(1)$. For $\bar{\phi}$ we set $\bar{\phi}^2 \sim \mathcal{O}(\alpha/\lambda)$, where $\lambda \leq \alpha \leq 1$ is to be determined self-consistently.

Note that the diagonalisation of ρ_t occurs in time as an *exponential* of an *exponential*. As a result, decoherence occurs extremely quickly, but not so quickly that $\mu t \ll 1$. Consequently, in order to evaluate the decoherence time in our model, we have to use Eq. (21). On taking the equality in (24) we find

$$\exp[2\mu t_D] \approx \frac{\lambda\sqrt{N}\bar{g}}{\bar{g}^2\alpha} = \mathcal{O}\left(\frac{\sqrt{N}}{\alpha}\right), \tag{25}$$

whereby

$$\mu t_D \simeq \frac{1}{4}\,\ln N - \frac{1}{2}\ln\alpha \simeq \ln\left(\frac{\eta}{T_{\rm c}\sqrt{\alpha}}\right).$$

This is an *upper* bound on t_D, but probably still qualitatively correct. The value of α is determined as $\alpha \simeq \sqrt{\mu/T_{\rm c}}$ from the condition that, at time t_D, $\langle|\phi|^2\rangle_t \sim \alpha\eta^2$. Since $\alpha \ll 1$, in principle, the field has not diffused far from the top of the hill before it is behaving classically.

For comparison, we find $t_{\rm sp}$, for which $\langle\phi^2\rangle_t \sim \eta^2$, given by

$$\exp[2\mu t_{\rm sp}] \approx \mathcal{O}\left(\frac{\eta^2}{\mu T_{\rm c}}\right). \tag{26}$$

The exponential factor, as always, arises from the growth of the unstable long-wavelength modes. The factor T_c^{-1} comes from the $\coth(\beta\omega/2)$ factor that encodes the initial Boltzmann distribution at temperature $T \gtrsim T_c$. As a result,

$$\mu t_{sp} \sim \ln\left(\frac{\eta}{\sqrt{\mu T_c}}\right), \tag{27}$$

whereby $1 < \mu t_D \leq \mu t_{sp}$, with

$$\mu t_{sp} - \mu t_D \simeq \frac{1}{4}\ln\left(\frac{T_c}{\mu}\right) > 1, \tag{28}$$

for weak enough coupling, or high enough initial temperatures (we have taken $T_0 \sim T_c$ throughout). This is our main result, that for the physically relevant modes (with small k_0) classical behavior has been established before the spinodal time, when the ground states have became populated.

3.5. Back-Reaction

We can now justify our earlier assumption that for an instantaneous quench, nonlinear behavior only becomes important just before the spinodal time (Karra and Rivers, 1997). To see this, we adopt the Hartree approximation, in which the equations of motion are linearized so that the field modes $f_k^{\pm}$ now satisfy the equation

$$\left[\frac{d^2}{dt^2} + \mathbf{k}^2 - \mu^2(t)\right] f_k^{\pm}(t) = 0,$$

where (Boyanovsky *et al.*, 1994, 1995; Ramsey and Hu, 1997)

$$-\mu^2(t) = m^2(t) + 4\lambda \int \frac{d^3p}{(2\pi)^3}\, C(p)[\, f_p^+(t)\, f_p^-(t) - 1] \tag{29}$$

and $C(p) = (\coth \beta\omega/2)/2\omega$, $\omega^2 = p^2 + m^2$.

A similar result would be obtained by extending our initial $O(2)$ theory to an $O(M)$ theory in the large-M limit before taking $M = 2$.

Our coarse-graining retains only the unstable modes in the integral, which suggests (Karra and Rivers, 1997) the approximate hybrid self-consistent equation for $\mu^2(t)$,

$$\mu^2(t) \simeq \mu^2 - C\lambda \frac{T\mu}{(\mu t_{sp})^{3/2}} \exp\left(2\int_0^t dt'\, \mu(t')\right) \tag{30}$$

($C = O(1)$), which has the exponential growth of the WKB solution, but non-singular behavior when $\mu(t) \approx 0$. The exact solution to Eq. (30) for $t < t_{sp}$ is $\mu(t) = \mu \tanh \mu(t_{sp} - t)$, irrespective of the values of the temperature T and the

coupling strength. In fact, we anticipated this in (27), when we estimated t_{sp} on the assumption that the back-reaction would only take effect in the final moments.

That is, the theory only ceases to behave like a free Gaussian theory with upside-down potential at a time t_B,

$$t_{sp} - t_B = O(\mu^{-1}). \tag{31}$$

It follows that $t_B \geq t_D$ in our ordering of scales $T_c \gg \mu$, but in practice T_c needs to be at least an order of magnitude larger than μ for this to be the case.

When (28) is valid, we see that ρ_r becomes diagonal before nonlinear terms could be relevant. In this sense, classical behavior has been achieved before quantum effects could destroy the positivity of the Wigner function W_r.

4. FINAL REMARKS

We have shown that in our model, ρ_r becomes diagonal before the spinodal time at which the order parameter field has first populated the ground-state values of the theory. Further, it can also be diagonal before nonlinear terms are relevant. That is, decoherence can be achieved before quantum effects destroy the positivity of the Wigner function W_r. Really, our t_D sets the time after which we have a classical probability distribution (positive definite) even for times $t > t_{sp}$. The existence of the environment is crucial in doing this. Of course, for non-Gaussian or delocalized (in the field space) initial states, it is clear that W_r will be nonpositive definite even in the linear regime, and therefore t_D should be smaller than the one we evaluated here. In the present work, t_D is the *classicalization time*-bound.

This result goes in the direction of justifying the use of classical numerical simulations for the analysis of the dynamics of the long-wavelengths modes after the quench.

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